Problem:

Part 1: The Locker Problem

Imagine you are at a school that still has student lockers. There are 1000 lockers, all shut and unlocked, and 1000 students. Here's the problem:

- 1. Suppose the first student goes along the row and opens every locker.
- 2. The second student then goes along and shuts every other locker beginning with number 2.
- 3. The third student changes the state of every third locker beginning with number 3 (If the locker is open the student shuts it, and if the locker is closed the student opens it).
- 4. The fourth student changes the state of every fourth locker beginning with number 4.

Imagine that this continues until the thousand students have followed the pattern with the thousand lockers. At the end, which lockers will be open and which will be closed? Why?

Now look at only the first 100 lockers. Which locker was switched the most times? How many lockers, and which ones, were touched exactly twice? Why?

Solution

The lockers that will be open are all the perfect squares between 1-1000

The lockers that were touched the most times are the ones with the most number of factors (12 factors)

The lockers that were only touched twice would be the locker numbers consisting of a prime number. Since these numbers only have 2 factors, 1 and itself, then we know it can only be touched exactly 2 times.

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