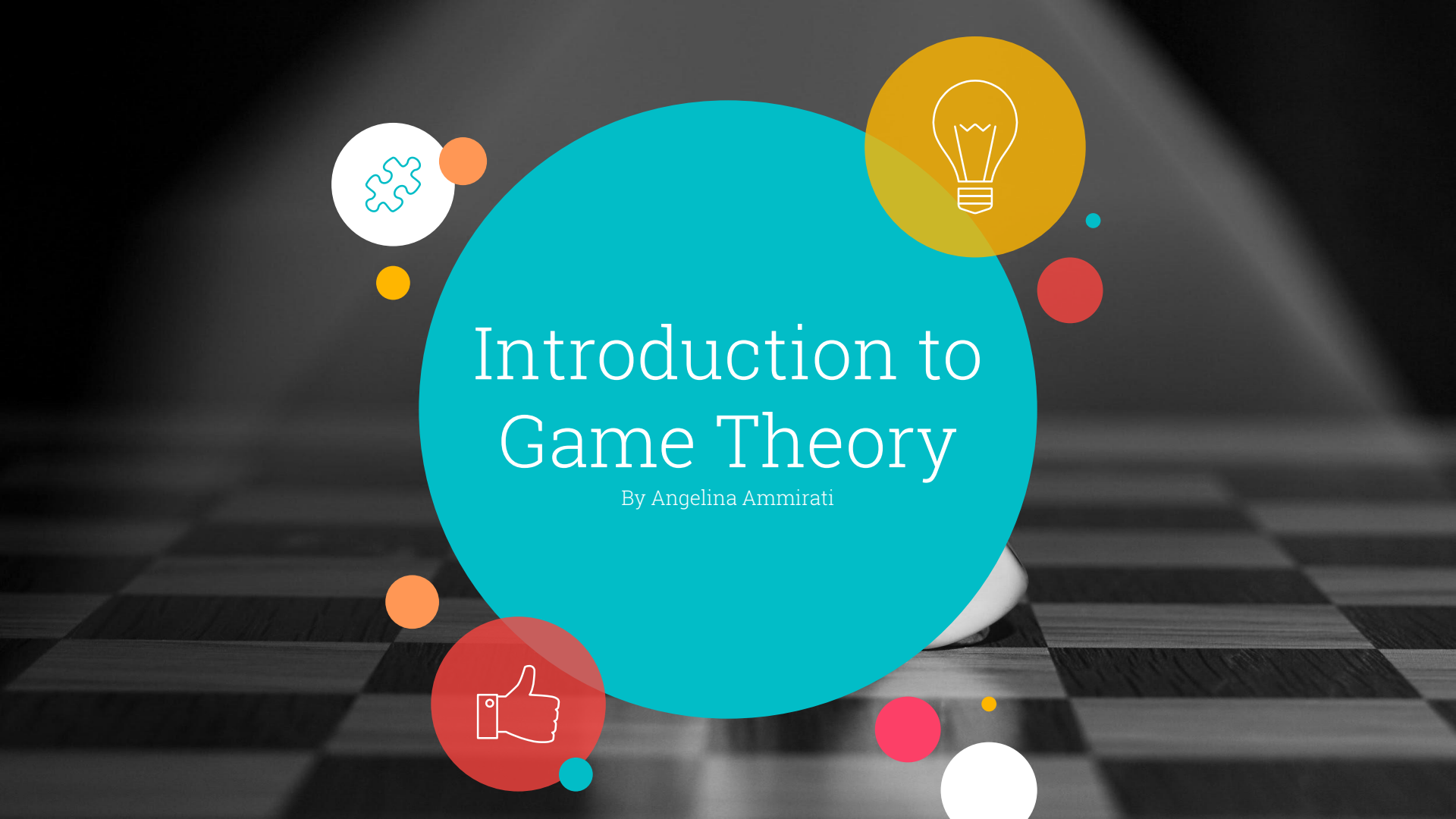

Introduction to Game Theory

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Game Theory

the general mathematical theory behind all social interactions and decisions

Background

John von Neumann, a mathematician, physicist, & computer scientist with an interest in poker. He wanted to create a general theory that could be applied to diplomacy, war, love, evolution, or business strategy to create a path of mathematics for life. John collaborated with Oskar Morgenstern, an economist, to write a book together in 1944 in which they claimed any economic situation could be defined as the outcome of a game between two or more players.

Definitions

- Players – share common knowledge of rules, available strategies, and possible payoffs
- Strategies – actions players take in a game
- Payoffs – outcome of the strategy applied by the player
  - (some examples could be profits, peace treaty, or a great deal on a car)
- Coalition – group of players in a cooperative game
- Shapley Value – a method of dividing up gains or costs among players according to the value of their individual contributions

Types of Games

Cooperative

- Every player has agreed to work together towards a common goal
  - Example: Splitting the bill at a restaurant
- Main Question: “How much should each player contribute to the coalition, and how much they should benefit from it”

Competitive

- Social interaction where there are winners and losers
  - Example: Chess, Poker, Simple election
- Main Thought: “What is good for me is bad for my opponent, and vice versa”

Note:

- ★ In a competitive situation, game theory can tell you how to be smart

- ★ In a cooperative situation, game theory can tell you how to be fair

The Prisoner Dilemma

Problem: Jack and Tom have been captured for robbing a bank. The police do not have enough evidence to convict them, but know they have committed the crime. They put Jack and Tom in separate interrogation rooms and lay out the consequences:

- If both Jack and Tom confess, they will each get 10 years in prison
- If one confesses and the other doesn't, the one who confessed will go free and the other will be sentenced to 20 years in prison
- If neither confess, they will both get 5 years for a different crime they were wanted for

Both prisoners are in separate interrogation rooms and will not be released until they have made a decision.

What do you think the best option for Jack and Tom is?

Tom	Jack	
	Silent	Confess
Silent	5 years 5 years	0 years 20 years
Confess	0 years 20 years	10 years 10 years

Solution

The best option for both Tom and Jack to choose is actually to both confess. According to what is considered the “dominant strategy” or common solution of the strategy that has the best payoff no matter what the other player(s) choose(s). John Nash, a mathematician in the 1950s, created a key concept of this prisoner dilemma to be applied to game theory known as the Nash Equilibrium. This is when a player in a game has made a choice that leaves them better off no matter what their opponent(s) decide to do.

Other types of games

One Person Games

Also known as games against nature

The player lists only available options and chooses the optimal outcome

Example: a person deciding whether to carry an umbrella weighs the costs and benefits of carrying or not carrying it.

Two-Person Constant-Sum Games

Perfect: by making use of all available information, the players can deduce strategies that are optimal, which makes the outcome preordained (strictly determined)

Example: Chess, Checkers

Imperfect: By choosing a strategy associated with what's known as the "saddlepoint" outcome, each player obtains an amount at least equal to his payoff at that outcome, no matter what the other player does. This payoff is called the value of the game

Two-Person Variable-Sum Games

The players have both common and opposed interests

Example: a buyer and a seller are engaged in a variable-sum game (the buyer wants a low price and the seller a high one, but both want to make a deal)

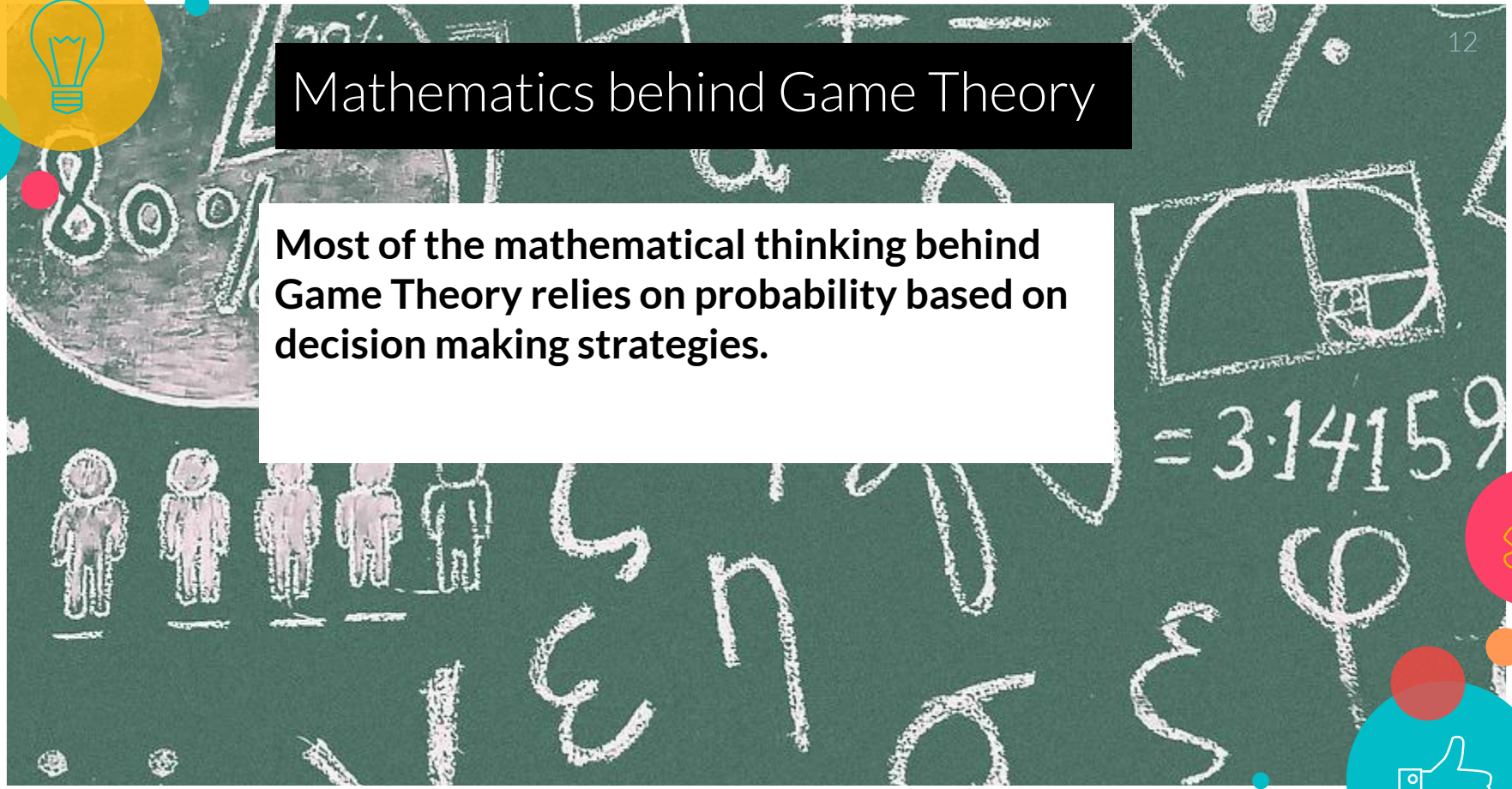
Other types of games

N-Person Games

players are not allowed to communicate and make binding agreements

imagine three players, A, B, and C, at the corners of an equilateral triangle. Each player has a gun with one bullet. Assume that each player is a perfect shot and can kill one other player at any time. There is no fixed order of play, but any shooting that occurs is sequential: no player fires at the same time as any other. Consequently, if a bullet is fired, the results are known to all players before another bullet is fired.

Mathematics behind Game Theory



Most of the mathematical thinking behind Game Theory relies on probability based on decision making strategies.

Reference

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